Example 1:

Student Car Age You want to prove that most students have cars that are not new (the car is older than 5 years). You get a random sample of 100 students and find that the average age of their cars is 8 years with a sample standard deviation of 2.5. If you wanted to be sure your numbers were accurate 95% of the time you'd create a confidence interval and check your data against it.

Step 1. What info do we have so far?

n = 100

$\bar{x} = 8$

$s = 2.5$

Using a 95% confidence

Step 2 What does the formula look like:

$$\bar{x} \pm E$$

$$or$$

$$\bar{x} - E < \mu < \bar{x} + E$$

where $$E = Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

AKA the formula should look like this:

$$\bar{x} - Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{x} + Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

where your answer is :

smaller number < μ < bigger number

where the smaller number is what you get from using the formula:

$$\bar{x} - Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

and the larger number is what you get from:

$$\bar{x} + Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

Step 3 Find your Z:

First of all $Z_{\frac{\alpha}{2}}$ only means that there are 2 tails and the % is in the middle. (So you'll have some really old cars and some really young ones.)

To find the Z find the middle % in the center of the bell curve using the Z table (standard normal table) or the ti calculator.

Since we're looking for the middle 95% we need to find the closest value to 0.0250 in the inside of the Z table or the calculator.

> Calculator instructions: 2^{nd} VARS 3:invnorm (value of the tail in decimal form)

You should get -1.96 for the bottom 2.5% and 1.96 for the top 2.5% So your $Z_{\alpha/2} = 1.96$

Step 4 Now you can plug all of your values into the equation:

$$\bar{x} - Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{x} + Z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$\bar{x} - 1.96\left(\frac{2.5}{\sqrt{100}}\right) < \mu < \bar{x} + 1.96\left(\frac{2.5}{\sqrt{100}}\right)$$

$$8 - 1.96\left(\frac{2.5}{\sqrt{100}}\right) < \mu < 8 + 1.96\left(\frac{2.5}{\sqrt{100}}\right)$$

$$8 - .49 < \mu < 8 + .49$$

$$7.51 < \mu < 8.49$$

Thus we can say that we're 95% confident that the population mean should be between 7.51 and 8.49.

Note:
If you are asked for the error (E) you would use E = 0.49 from the formula:

$$E = 1.96\left(\frac{2.5}{\sqrt{100}}\right) = 0.49$$