Basics of the Central Limit Theorem

The theory is a combination of the LAW OF LARGE NUMBERS and what we know about the normal curve. This shows that the mean and standard deviations can be approximated using the mean of the sample means and the standard deviation of the sample means.

Formulas:

$$\mu_{\bar{x}} = \mu \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Example explaining why this works:

A small apartment building has 3 apartments. Apartment A has 1 person living there, apartment B has 2 people living there, and apartment C has 5 people living there.

Find the mean and standard deviation of the population of the building

μ = _____ σ = _____

Now to prove the formula with a sample of 2 apartments:

—We need to form all samples of size 2, using replacement since the population is very small.

—Then we find the sample mean for each sample of 2 apartments.

—Find μ and σ

Apartments	A,A	A,B	A,C	B,A	B,B	B,C	C,A	C,B	C,C
# of people in each apartment	1,1	1,2	1,5	2,1	2,2	2,5	5,1	5,2	5,5
Average of samples	1	1.5	3.5	1.5	2	3.5	3	3.5	5

Find the mean and standard deviation of the sample means of the building

μ = _____ σ = _____

Example:

If μ = 40, σ = 10 and n = 4, find the mean and standard deviation of the sample means:

μ = _____ σ = _____

If μ = 40, σ = 10 and n = 25, find the mean and standard deviation of the sample means:

μ = _____ σ = _____

Distribution of the sample means

—If the population is normally distributed, then the sample means will be normally distributed.

—If the population is not normally distributed, then the sample means will be normally distributed if the sample size is at least 30.

—If we take samples of size n from some population, under the previous conditions, then we can determine the probability of the sample means fulfilling some condition. We use the z formula with the new values for the standard deviation and the means.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

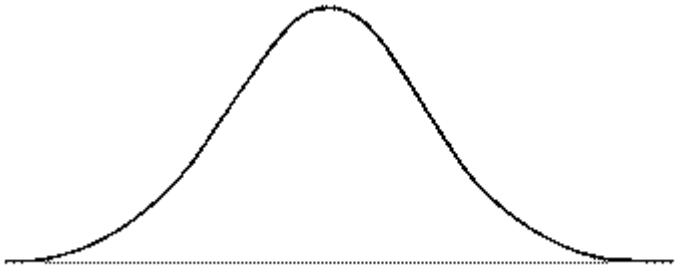
Example #1

The heights of kindergarten children are approximately normally distributed with a mean of 39 and a standard deviation of 2.

A.) If one child is randomly selected, what is the probability that the child is taller than 41 inches?

Note: This is 1 child – Not the Central Limit Theorem!

X = _____ μ = _____ σ = _____



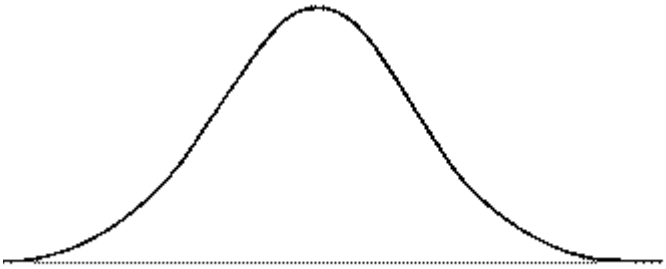
Z = _____

P( ) = _____

B.) Suppose we have a class of 30 kindergarten children. What is the probability that the mean height of these children exceeds 41 inches?

Note: This is the Central Limit Theorem as it is asking about the probability of a sample mean!

$\bar{X}$ = _____ μ = _____ σ = _____ n = _____



Z = _____

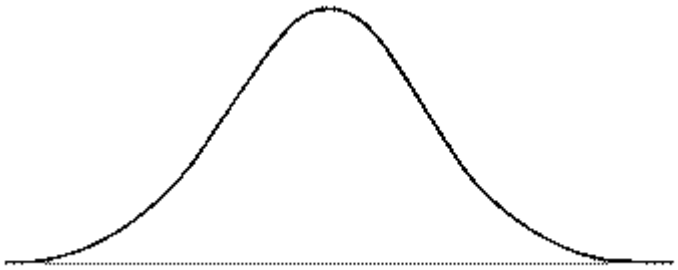
P( ) = _____

Another Example

The IQ scores of adults are normally distributed with a mean of 100 and a standard deviation of 15.

A.)If one person is randomly selected, what is the probability that his IQ will be greater than 110?

X = _____ μ = _____ σ = _____

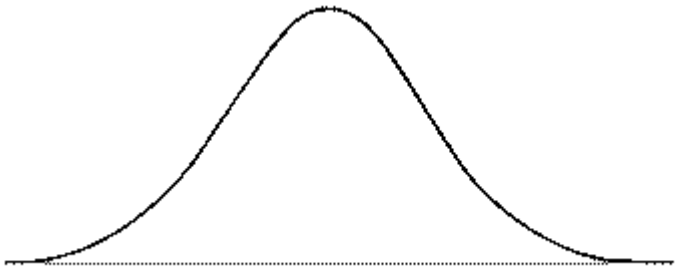


Z = _____

P( ) = _____

B.) If ten people are randomly selected, what is the probability that mean of those 10 people's IQ will be greater than 110?

$\bar{X}$ = _____ μ = _____ σ = _____ n = _____



Z = _____

P( ) = _____

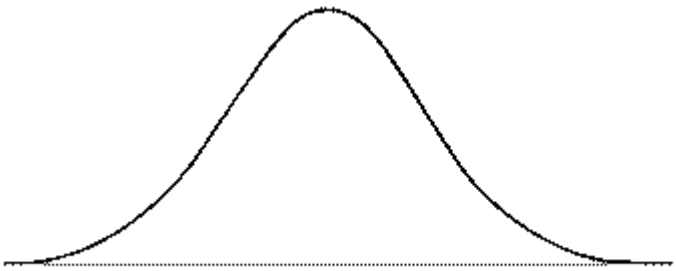
Your Turn

The means and standard deviations of the heights of female college students are as follows:

μ = 64″ σ = 2.75″

If Prof. K's favorite Disney character is Stitch. What is the probability that the average height of any 5 female students are short enough (less than 59 inches tall) to play Stitch in Walt Disney World?

$\bar{X}$ = _____ μ = _____ σ = _____ n = _____



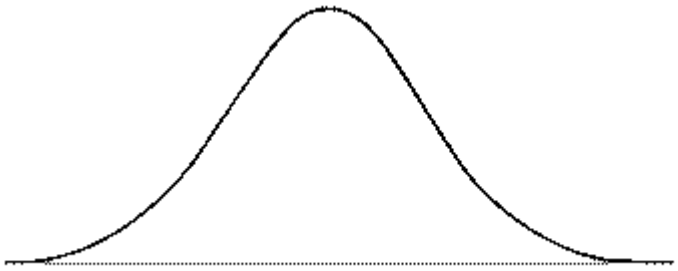
Z = _____

P( ) = _____

NOTE: Some professors prefer using the following notation to show the mean and standard deviation: $N\sim(\mu, \sigma)$ In other words a normal distribution with a mean of 0 and a standard deviation of 1 can be written as $\sim N(0,1)$

The population mean and standard deviation of the heights of adults are as follows:
Male: μ = 69″ σ = 2.90″
Female: μ = 64″ σ = 2.75″
Pick your favorite Disney character. What is the probability that the average height of any 30 students is short enough to play your favorite character in Walt Disney World?

$\bar{X}$ = _____ μ = _____ σ = _____ n = _____



Z = _____

P( ) = _____

Disney Character Height Requirements

Princesses:
Rapunzel, Cinderella, Aurora, Belle, Ariel, Snow White, Merida, Jasmine, Anna, and Elsa: 5'3″ - 5'7″
Mulan: 5'2″ - 5'6″
Tiana: 5'5″ - 5'8″

Fairies:
Tinker Bell: 4'10″ - 5'2″
Iridessa, Rosetta, Silvermist, Fawn, Vidia, and Periwinkle: 5'2″ – 5'5″
Fairy Godmother: 5'4″ - 5'6″

Villains:
Wicked Step-Mother, Step-Sisters, Malefacent: 5'6″ - 5'10″

Other Women:
Mary Poppins: 5'5″ - 5'8″
Alice and Wendy: 5'2″ - 5'4″

Princes:
Prince Charming, Prince Eric, Aladdin, Prince Naveen, Snow White's Prince, Flynn Rider, and the Beast in human form: 5'10″ - 6'0″

Male Fairies:
Terrence: 5'4″ - 5'7″

Male Villains:
Dr. Facilier: 6'0″ - 6'6″
Gaston: 5'11″ - 6'3″

Other Men:
Peter Pan: 4'10″ - 5'2″
Mad Hatter: 5'4″ - 5'6″
Jack Sparrow, Bert: 5'8″ - 6'1″

Fur/Masked Characters No specific gender:
Mickey Mouse and Minnie Mouse: 4'8″ - 5'2″
Daisy and Donald: 4'6″ – 4'10″
Chip and Dale: 5'2″ - 5'4″
Pluto: 5'6″ - 5'8″
Goofy: 6″0″ – 6'3″
Huey, Dewey, Louie: 4'0″ - 4'6″
Lilo and Stitch: 4'9″ – 4'11″
Dopey: 4'8″ – 4'10″
Piglet: 4'10″ - 5'0″
White Rabbit, Pooh: 5'0″ - 5'2″
Suzy, Perla, Jessie, and Bullseye: 5'2″ - 5'4″
Rafiki, Flick, Buzz Lightyear: 5″6″ – 5'8″
Eeyore: 5'7″ – 5'9″
Tigger: 5'10″ – 6'0″
Green Army Men: 5'11″ - 6'0″
Captain Hook, Genie: 6'0″ - 6'2″
Beast, Woody, Jafar: 6'2″ - 6'4″