Rare Event Rule for Inferential Statistics:
If, under a given assumption (such as a lottery being fair), the probability of a particular observed event (such as five consecutive lottery wins) is extremely small, we conclude that the assumption is probably not correct.

Terminology and notation
Event = Any collection of results or outcomes of a procedure
Simple Event =An outcome or an event that cannot be further broken down into simpler components.
Sample Space = Consists of all possible *simple* events. That is, the sample space consists of all outcomes that cannot be broken down any further.
P = a probability.
A, B, and *C* (Any capital letter) = specific events.
P (*A*) = the probability of event A occurring.

Rule 1: Relative Frequency Approximation of Probability
Conduct (or observe) a procedure a large number of times, and count the number of times event *A* actually occurs. Based on these actual results, *P(A)* is *estimated* as follows:

$$P(A) = \frac{\text{number of times A occurred}}{\text{number of times procedure was repeated}} = \frac{f}{n}$$

Rule 2: Classical Approach to Probability
Assume that a given procedure has *n* different simple events and that each of those simple events has an *equal chance* of occurring. If event *A* can occur in *s* of these *n* ways, then:

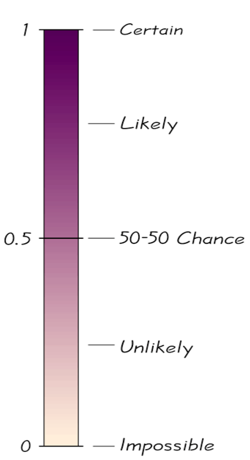
$$P(A) = \frac{\text{number of ways A can occur}}{\text{number of ways simple events can occur}} = \frac{S}{n}$$

Rule 3: Subjective Probabilities
P(*A*), the probability of event *A*, is found by simply guessing or estimating its value based on knowledge of the relevant circumstances.

Law of large numbers
As a procedure is repeated again and again, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability.

Example: Roulette You plan to bet on number 13 on the next spin of a roulette wheel. What is the probability that you will lose?

Solution: A roulette wheel has 38 different slots, only one of which is the number 13. A roulette wheel is designed so that the 38 slots are equally likely. Among these 38 slots, there are 37 that result in a loss. Because the sample space includes equally likely outcomes, we use the classical approach (Rule 2) to get P(Loss)= 37/38



Probability limits(Picture to the left):
The probability of an impossible event is 0.
The probability of an event that is certain to occur is 1.
$0 \le P(A) \le 1$ for any event *A*.

Compliments definition: The complement of event *A*, denoted by *A*, consists of all outcomes in which the event *A* does *not* occur.

Example: In reality, more boys are born than girls. In one typical group, there are 205 newborn babies, 105 of whom are boys. If one baby is randomly selected from the group, what is the probability that the baby is *not* a boy?

Solution: Because 105 of the 205 babies are boys, it follows that 100 of them are girls, so
P(not selecting a boy) = *P*(boy) = *P*(girl) = 100/205 = 0.488

Rounding Rules:
When expressing the value of a probability, either give the *exact* fraction or decimal or round off final decimal results to three significant digits. (*Suggestion*: When the probability is not a simple fraction such as 2/3 or 5/9, express it as a decimal so that the number can be better understood.)

Compound Event
Any event combining 2 or more simple events

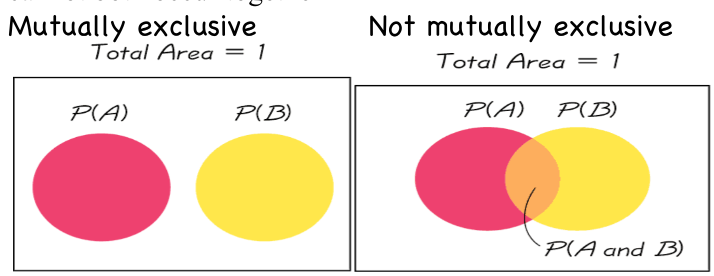
Notation
P(*A* or *B*) =
P(event *A* occurs or event *B* occurs or they both occur)

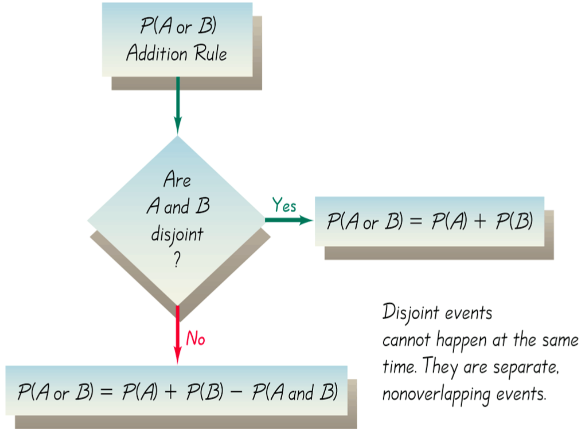
General rule for finding a compound event:
When finding the probability that event *A* occurs or event *B* occurs, find the total number of ways *A* can occur and the number of ways *B* can occur, but *find the total in such a way that no outcome is counted more than once.*

Formal Addition Rule
P(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*)
where *P*(*A* and *B*) denotes the probability that *A* and *B* both occur at the same time as an outcome in a trial or procedure.

Intuitive Addition Rule
To find *P*(*A* or *B*), find the sum of the number of ways event *A* can occur and the number of ways event *B* can occur, *adding in such a way that every outcome is counted only once*. *P*(*A* or *B*) is equal to that sum, divided by the total number of outcomes. In the sample space.

Events *A* and *B* are disjoint (or mutually exclusive) if they cannot both occur together





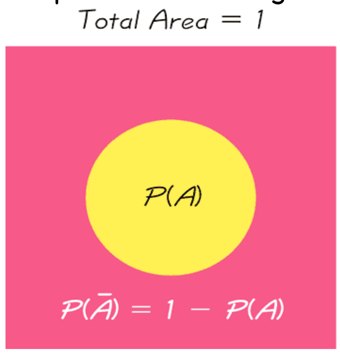
Complimentary events: $P(A)$ and $P(\bar{A})$ are mutually exclusive if all simple events are either in A or $\bar{A}$.
Formula:

$P(A) + P(A) = 1$

$= 1 - P(A)$

$P(A) = 1 - P(A)$

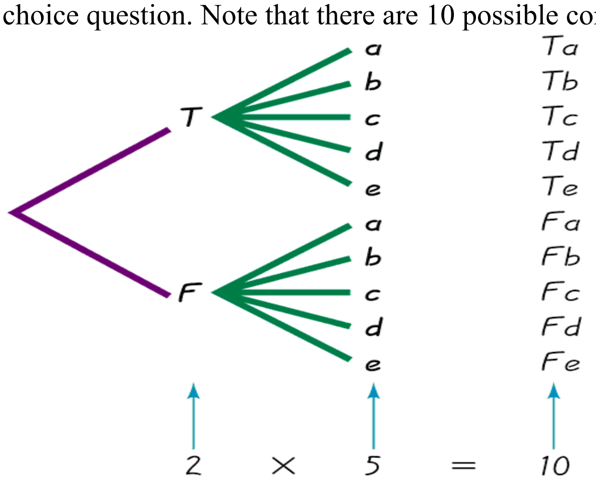
Compliment venn diagram:



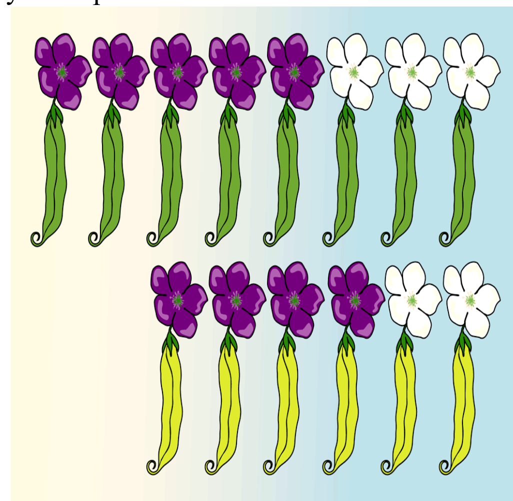
Notation:
$P(A \text{ and } B) =$
$P(\text{event } A \text{ occurs in a first trial and event } B \text{ occurs in a second trial})$

Tree Diagram:
A tree diagram is a picture of the possible outcomes of a procedure, shown as line segments emanating from one starting point. These diagrams are helpful in counting the number of possible outcomes if the number of possibilities is not too large.

The possible outcomes for a true/false followed by a multiple choice question. Note that there are 10 possible combinations.



EXAMPLE: Genetics Experiment Mendel's famous hybridization experiments involved peas, like those shown in Figure 3-3 (below). If two of the peas shown in the figure are randomly selected *without replacement*, find the probability that the first selection has a green pod and the second has a yellow pod.



First selection: *P*(green pod) = 8/14 (14 peas, 8 of which have green pods)

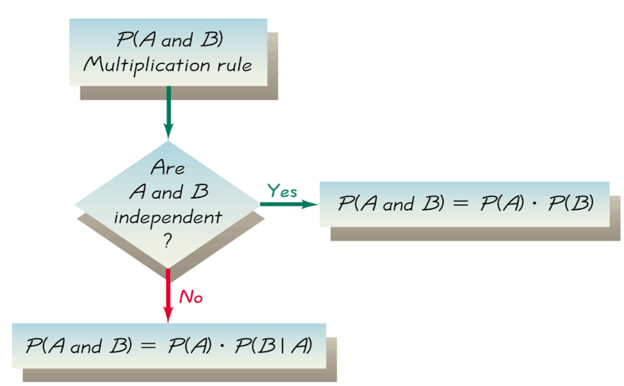
Second selection: *P*(yellow pod) = 6/13 (13 peas remaining, 6 of which have yellow pods)

With *P*(first pea with green pod) = 8/14 and *P*(second pea with yellow pod) = 6/13, we have *P*(First pea with green pod and second pea with yellow pod) = $\frac{8}{14} \cdot \frac{6}{13} \approx 0.264$
The example above illustrates the important principle that the *probability for the second event B should take into account the fact that the first event A has already occurred.*

Conditional probability and Multiplication Rules
Notation: $P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred (read $B|A$ as "B given A.")

Independent Events
Two events A and B are independent if the occurrence of one does not affect the probability of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the occurrence of the others.) If A and B are not independent, they are said to be dependent.
Formula: $P(A \text{ and } B) = P(A) \cdot P(B|A)$
Note that if A and B are independent events, P(B|A) is really the same as P(B)

Intuitive Multiplication Rule
When finding the probability that event A occurs in one trial and B occurs in the next trial, multiply the probability of event A by the probability of event B, but be sure that the probability of event B takes into account the previous occurrence of event A.



SUMMARY of multiplication and addition rules:
In the addition rule, the word "or" on *P*(*A* or *B*) suggests addition. Add *P*(*A*) and *P*(*B*), being careful to add in such a way that every outcome is counted only once.

In the multiplication rule, the word "and" in *P*(*A* and *B*) suggests multiplication. Multiply *P*(*A*) and *P*(*B*), but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.

Rules for finding At least one

"At least one" is equivalent to "one or more."
The complement of getting at least one item of a particular type is that you get no items of that type.

To find the probability of *at least one* of something, calculate the probability of *none*, then subtract that result from 1. That is, *P*(at least one) = 1 – *P*(none)

Example: Gender of Children Find the probability of a couple having at least 1 girl among 3 children. Assume that boys and girls are equally likely and that the gender of a child is independent of the gender of any brothers or sisters.
Solution:
Step 1: Use a symbol to represent the event desired.
In this case, let *A* = at least 1 of the 3 children is a girl.
Step 2: Identify the event that is the complement of *A*.
A = not getting at least 1 girl among 3 children
= all 3 children are boys
= boy and boy and boy
Step 3: Find the probability of the complement.

P(*A*) = *P*(boy and boy and boy) $= \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{8}$

Step 4: Find *P*(A) by evaluating 1 – *P*(*A*).

$$P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{8} = \frac{7}{8}$$

Interpretation of answer: There is a 7/8 probability that if a couple has 3 children, at least 1 of them is a girl.

Conditional probability

A conditional probability of an event is a probability obtained with the additional information that some other event has already occurred. *P*(*B* | *A*) denotes the conditional probability of event *B* occurring, given that *A* has already occurred, and it can be found by dividing the probability of events *A* and *B* both occurring by the probability of event *A*:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(B)}$$

If you have a table of values you can do this by finding the area that has both A and B and dividing that by the total of B.

	A	C	Totals
B	# = A and B	# = C and B	# = Total of B
D	# = A and D	# = C and D	# = Total of D
Totals	# = Total of A	# = Total of C	# = Total of totals

Intuitive Approach to Conditional Probability

The conditional probability of *B* given *A* can be found by assuming that event *A* has occurred and, working under that assumption, calculating the probability that event *B* will occur.

Testing for independence:

We already know that that events *A* and *B* are independent if the occurrence of one does not affect the probability of occurrence of the other. This suggests the following test for independence:

A and B are INDEPENDENT if:
P(B|A) = P(B)
or
P(A and P(B) = P(A)*P(B)
or
P(A)+P(B) –P(AorB) = P(A)*P(B)

A and B are DEPENDENT if:
P(B|A) ≠ P(B)
or
P(A and P(B) ≠ P(A)*P(B)
or
P(A)+P(B) –P(AorB) ≠ P(A)*P(B)